Aligators Demo

Note: Aligators.m relies on LoopSyn.m. LoopSyn.m automatically rewrites a nested loop conditionals over matrices into an equivalent sequence of unnested loops without conditionals over 1D arrays.

LoopSyn

In[4]:= << LoopSyn.m

LUdecomposition.GetL

In[5]:= loopSynInput = {
                      
                      
                      
                      
                      
                      }

Out[5]= 

In[6]:= LoopSyn[loopSynInput]

Starting LoopSyn for 3 loops with matrix indices (i, j)!

Loop Nr. 1

Guarded assg form: ((i > j), (L[i, j] = LU[i, j]))
Guard: (i > j)
Guard Polynomial Invariant: c$1521 + i - j
Body: (L[i, j] = LU[i, j])

Creating the loop body with assignments over (i, j) such that c$1521 + i - j=0 is a loop invariant. Loop condition is z<n^2.
If `-1 + $l$' is a natural number, then:

If `-1 + $l$' is a natural number, then:

Loop Assignments over $(i, j)$:

(i = i + $3 + j $4, j = j + $2)

Recurrence System:

\{i[1 + $l] = $3 + i[$l] + $4 j[$l], j[1 + $l] = $2 + j[$l]\}

General Closed Form System:

\{i = i[0] + \frac{1}{2} $1 (2 $3 + $4 ((-1 + $l) $2 + 2 j[0])), j = $1.$2 + j[0]\}

Branch Guard's Closed Form:

$c$\$1521 - $1 $2 + i[0] - j[0] + \frac{1}{2} $1 (2 $3 + $4 ((-1 + $l) $2 + 2 j[0]))$

Constraint System on Unknown Coefficients:

\{($3 \rightarrow $2, i[0] \rightarrow -c$1521 + j[0], $4 \rightarrow 0), ($3 \rightarrow -$4 j[0], i[0] \rightarrow -c$1521 + j[0], $2 \rightarrow 0)\}

Additional Constraints on the Unknown Coefficients:

($3^2 + $4^2 \neq 0, $2 \neq 0$)

Joint Constraint System:

$3 = $2 && i[0] = -c$1521 + j[0] && $4 = 0$

Joint Constraint System:

$3 = -$4 j[0] && i[0] = -c$1521 + j[0] && $2 = 0$

Suitable Loop Constraint Solution:

\{($3 \rightarrow $2, i[0] \rightarrow -c$1521 + j[0], $4 \rightarrow 0)\}

with initial value:

\{j[0] \rightarrow 0\}

Parameter Basis Solution Choice:

($2 \rightarrow 1$)

Loop Body:

\{i = 1 + i, j = 1 + j\}

Initial Value Assignments:

\{i = -c$1521, j = 0\}

Assignment of array variable $z$:

\{$z = 1 + n + z$\}

Initial value assignment of array variable $z$:

\{$z = (-1 - c$1521) n\}

Corresponding While Loop’s

Body: (i = 1 + i, j = 1 + j, z = 1 + n + z)
Condition: \(z < n^2\)

Initial Value Assignments:\(i = -c\theta 1521, j = 0, z = (-1 - c\theta 1521) n\)

Loop Nr. 2

Guarded assg form: \([i = j], [L[i, j] = 1]\)
Guard: \(i = j\)
Guard Polynomial Invariant: \(i - j\)
Body: \([L[i, j] = 1]\)

Creating the loop body with assignments over \((i, j)\)
such that \(i - j = 0\) is a loop invariant.
Loop condition is \(z < n^2\).

If \(-1 + $5' is a natural number, then:

If \(-1 + $5' is a natural number, then:

Loop Assignments over \((i, j)\):
\((i = i + $7 + j $8, j = j + $6)\)

Recurrence System:
\([i[1 + $5] = $7 + i[0] + $8 j[0] + j[1 + $5] = $6 + j[0]]\)

General Closed Form System:
\(\left\{ i = i[0] + \frac{1}{2} $5 (2 $7 + $8 ((-1 + $5) $6 + 2 j[0])), j = $5 $6 + j[0]\right\}\)

Branch Guard's Closed Form:
\(-$5 $6 + i[0] - j[0] + \frac{1}{2} $5 (2 $7 + $8 ((-1 + $5) $6 + 2 j[0]))\)

Constraint System on Unknown Coefficients:
\(\left\{ [i[7] = $6, i[0] = j[0], $8 = 0], [i[7] + $8 j[0], i[0] = j[0], $6 = 0]\right\}\)

Additional Constraints on the Unknown Coefficients:
\(\left\{ $7^2 + $8^2 \neq 0, $6 \neq 0\right\}\)

Joint Constraint System:
\(\left\{ $7 = $6 \& i[0] = j[0] \& $8 = 0\right\}\)

Joint Constraint System:
\(\left\{ $7 = -$8 j[0] \& i[0] = j[0] \& $6 = 0\right\}\)

Suitable Loop Constraint Solution:
\(\left\{ $7 \rightarrow $6, i[0] \rightarrow j[0], $8 \rightarrow 0\right\}\)

with initial value:
\(\left\{ i[0] \rightarrow 0\right\}\)
Parameter Basis Solution Choice:
\( (\delta \rightarrow 1) \)

Loop Body:
\( (i = 1 + i, j = 1 + j) \)

Initial Value Assignments:
\( (i = 0, j = 0) \)

Assignment of array variable \( z \):
\( (z = 1 + n + z) \)

Initial value assignment of array variable \( z \):
\( (z = -n) \)

Corresponding While Loop's
Body:
\( (i = 1 + i, j = 1 + j, z = 1 + n + z) \)

Condition:
\( (z < n^2) \)

Initial Value Assignments:
\( (i = 0, j = 0, z = -n) \)

Loop Nr. 3

Guarded assg form: \( (i < j), (L[i, j] = 0) \)
Guard: \( (i < j) \)
Guard Polynomial Invariant:
\( c\$1721 + i - j \)
Body:
\( (L[i, j] = 0) \)

Creating the loop body with assignments over \( (i, j) \) such that \( c\$1721 + i - j = 0 \) is a loop invariant.
Loop condition is \( z < n^2 \).

If \( '-1 + 9' \) is a natural number, then:
If \( '-1 + 9' \) is a natural number, then:

Loop Assignments over \( (i, j) \):
\( (i = i + 11 + j \cdot 12, j = j + 10) \)

Recurrence System:
\( (i[1 + 9] = 11 + i[9] + 12 \cdot j[9], j[1 + 9] = 10 + j[9]) \)

General Closed Form System:
\( (i = i[0] + \frac{1}{2} \cdot 9 \cdot (2 \cdot 11 + 12 \cdot (10 \cdot (-1 + 9) + 2 \cdot j[0])), j = 10 \cdot 9 + j[0]) \)

Branch Guard's Closed Form:
\( c\$1721 - 10 \cdot 9 + i[0] - j[0] + \frac{1}{2} \cdot 9 \cdot (2 \cdot 11 + 12 \cdot (10 \cdot (-1 + 9) + 2 \cdot j[0])) \)
Constraint System on Unknown Coefficients:
($(11 \rightarrow 10, i[0] \rightarrow -c\$1721 + j[0], 12 \rightarrow 0), (11 \rightarrow -12 \cdot j[0], i[0] \rightarrow -c\$1721 + j[0], 10 \rightarrow 0))$

Additional Constraints on the Unknown Coefficients:
($(11^2 + 12^2 \neq 0, 10 \neq 0)$

Joint Constraint System:
$11 = 10 \& \& i[0] = -c\$1721 + j[0] \& \& 12 = 0$

Joint Constraint System:
$11 = -12 \cdot j[0] \& \& i[0] = -c\$1721 + j[0] \& \& 10 = 0$

Suitable Loop Constraint Solution:
($(11 \rightarrow 10, i[0] \rightarrow -c\$1721 + j[0], 12 \rightarrow 0)$
with initial value:
$
\{ j[0] \rightarrow 0 \}$

Parameter Basis Solution Choice:
$(10 \rightarrow 1)$

Loop Body:
$
\{ i = 1 + i, j = 1 + j \} \$

Initial Value Assignments:
$\{ i = -c\$1721, j = 0 \}$

Assignment of array variable z:
(z = 1 + n + z)

Initial value assignment of array variable z:
(z = (-1 - c\$1721) n)

Corresponding While Loop's
Body:(i = 1 + i, j = 1 + j, z = 1 + n + z)

Condition:(z < n^2)

Initial Value Assignments:(i = -c\$1721, j = 0, z = (-1 - c\$1721) n)

Out[6] = 
$\{ \{ i = -c\$1521, j = 0, z = (-1 - c\$1521) n \}, \{ z < n^2 \}, \{ i = 1 + i, j = 1 + j, z = 1 + n + z \} \},$

$\{ \{ i = 0, j = 0, z = -n \}, \{ z < n^2 \}, \{ i = 1 + i, j = 1 + j, z = 1 + n + z \} \}$,

$\{ \{ i = -c\$1721, j = 0, z = (-1 - c\$1721) n \}, \{ z < n^2 \}, \{ i = 1 + i, j = 1 + j, z = 1 + n + z \} \}$
QRdecomposition.GetR

\begin{verbatim}
In[7]:= loopSynInput = {
  { {i < j}, {R[i, j] = QR[i, j]}}},
  { {i = j}, {R[i, j] = Riag[i]}}},
  { {i < j}, {R[i, j] = 0}}
},
  { {i, 0, n}, {j, 0, n}}}

Out[7]= {{ { {i < j}, {R[i, j] = QR[i, j]}}},
  { {i = j}, {R[i, j] = Riag[i]}}},
  { {i < j}, {R[i, j] = 0}}},
  { {i, 0, n}, {j, 0, n}}}

In[8]:= LoopSyn[loopSynInput]

Starting LoopSyn for 3 loops with matrix indicees (i,j)!

Loop Nr. 1

Guarded assg form: ((i < j), (R[i, j] = QR[i, j]))
Guard: (i < j)
Guard Polynomial Invariant: c$1822$ + $i - j$
Body: (R[i, j] = QR[i, j])

Creating the loop body with assignments over \( (i, j) \)
such that c$1822$ + $i - j$=0 is a loop invariant.
Loop condition is \( z < n^2 \).

If '−1 + $\$13$' is a natural number, then:

If '−1 + $\$13$' is a natural number, then:

Loop Assignments over \( (i, j) \):
\( i = i + $\$15 + j$ $\$16, j = j + $\$14 \)

Recurrence System:
\( i[1 + $\$13$] = $\$15 + i[1 + $\$13$] + $\$16 j[1 + $\$13$], j[1 + $\$13$] = $\$14 + j[1 + $\$13$] \)

General Closed Form System:
\( i = i[0] + \frac{1}{2} $\$13 (2 $\$15 + $\$16 ((-1 + $\$13$) $\$14 + 2 j[0])), j = $\$13 $\$14 + j[0]) \)

Branch Guard's Closed Form:
\( c$1822$ - $\$13$ $\$14 + i[0] - j[0] + \frac{1}{2} $\$13 (2 $\$15 + $\$16 ((-1 + $\$13$) $\$14 + 2 j[0])) \)

Constraint System on Unknown Coefficients:
\( (($\$15 \rightarrow $\$14, i[0] \rightarrow -c$1822$ + j[0], $\$16 \rightarrow 0), ($\$15 \rightarrow -$\$16 j[0], i[0] \rightarrow -c$1822$ + j[0], $\$14 \rightarrow 0)) \)

Additional Constraints on the Unknown Coefficients:
\( ($\$15^2 + $\$16^2 \neq 0, $\$14 \neq 0) \)
\end{verbatim}
Joint Constraint System:
$15 = 14 \&\& i[0] = -c1822 + j[0] \&\& 16 = 0$

Joint Constraint System:
$15 = -14 j[0] \&\& i[0] = -c1822 + j[0] \&\& 14 = 0$

Suitable Loop Constraint Solution:
$(15 \rightarrow 14, i[0] \rightarrow -c1822 + j[0], 16 \rightarrow 0)$

with initial value:
$j[0] \rightarrow 0$

Parameter Basis Solution Choice:
$(14 \rightarrow 1)$

Loop Body:
\{i = 1 + i, j = 1 + j\}

Initial Value Assignments:
\{i = -c1822, j = 0\}

Assignment of array variable z:
\{z = 1 + n + z\}

Initial value assignment of array variable z:
\{z = (-1 - c1822) n\}

Corresponding While Loop's
Body:\{i = 1 + i, j = 1 + j, z = 1 + n + z\}

Condition:\{z < n^2\}

Initial Value Assignments:\{i = -c1822, j = 0, z = (-1 - c1822) n\}

Loop Nr. 2

Guarded assg form: \{(i = j), \{R[i, j] = Riag[i]\}\}

Guard: \{i = j\}

Guard Polynomial Invariant: i - j

Body: \{R[i, j] = Riag[i]\}

Creating the loop body with assignments over \{i, j\}
such that \(i - j = 0\) is a loop invariant.
Loop condition is \(z < n^2\).

If `-1 + $17$' is a natural number, then:

If `-1 + $17$' is a natural number, then:

Loop Assignments over \{i, j\}:
\{i = i + 19 + j 020, j = j + 18\}
Recurrence System:
\( \{ i[1 + j] = 19 + i[17] + 20 j[17], j[1 + j] = 18 + j[17] \} \)

General Closed Form System:
\( \{ i = i[0] + \frac{1}{2} \times 17 (2 \times 19 + 20 ((-1 + 17) \times 18 + 20 j[0])), j = 17 \times 18 + j[0] \} \)

Branch Guard's Closed Form:
\( -17 \times 18 + i[0] - j[0] + \frac{1}{2} \times 17 (2 \times 19 + 20 ((-1 + 17) \times 18 + 20 j[0])) \)

Constraint System on Unknown Coefficients:
\( \{(19 \rightarrow 18, i[0] \rightarrow j[0], 20 \rightarrow 0), (19 \rightarrow -20 j[0], i[0] \rightarrow j[0], 18 \rightarrow 0)\} \)

Additional Constraints on the Unknown Coefficients:
\( \{(19^2 + 20^2 \neq 0, 18 \neq 0)\} \)

Joint Constraint System:
\( 19 = 18 \&\& i[0] = j[0] \&\& 20 = 0 \)

Joint Constraint System:
\( 19 = -20 j[0] \&\& i[0] = j[0] \&\& 18 = 0 \)

Suitable Loop Constraint Solution:
\( \{(19 \rightarrow 18, i[0] \rightarrow j[0], 20 \rightarrow 0)\} \) with initial value:
\( \{(j[0] \rightarrow 0)\} \)

Parameter Basis Solution Choice:
\( (18 \rightarrow 1) \)

Loop Body:
\( \{i = 1 + i, j = 1 + j\} \)

Initial Value Assignments:
\( \{i = 0, j = 0\} \)

Assignment of array variable \( z \):
\( z = 1 + n + z \)

Initial value assignment of array variable \( z \):
\( z = -n \)

Corresponding While Loop's

Body: \( \{i = 1 + i, j = 1 + j, z = 1 + n + z\} \)

Condition: \( z < n^2 \)

Initial Value Assignments: \( \{i = 0, j = 0, z = -n\} \)

Loop Nr. 3
Guarded assg form: \([i < j], (R[i, j] = 0)\)
Guard: \(i < j\)
Guard Polynomial Invariant: \(c2021 + i - j\)
Body: \((R[i, j] = 0)\)

Creating the loop body with assignments over \((i, j)\) such that \(c2021 + i - j = 0\) is a loop invariant.
Loop condition is \(z < n^2\).

If \(-1 + 21' is a natural number, then:
If \(-1 + 21' is a natural number, then:

Loop Assignments over \((i, j)\):
\((i = i + 23 + j 24, j = j + 22)\)

Recurrence System:

General Closed Form System:
\([i = i[0] + \frac{1}{2} 21 \lbrace 2 \times 23 + 24 (\lbrace -1 + 21 \rbrace 22 + 2 j[0]) \rbrace, j = 21 \times 22 + j[0])\)

Branch Guard’s Closed Form:
\(c2021 - 21 22 + i[0] - j[0] + \frac{1}{2} 21 \lbrace 2 \times 23 + 24 (\lbrace -1 + 21 \rbrace 22 + 2 j[0]) \rbrace\)

Constraint System on Unknown Coefficients:
\(((23 \rightarrow 22, i[0] \rightarrow -c2021 + j[0], 24 \rightarrow 0), (23 \rightarrow -24 j[0], i[0] \rightarrow -c2021 + j[0], 22 \rightarrow 0))\)

Additional Constraints on the Unknown Coefficients:
\((23^2 + 24^2 \neq 0, 22 \neq 0)\)

Joint Constraint System:
\(23 = 22 \land i[0] = -c2021 + j[0] \land 24 = 0\)

Joint Constraint System:
\(23 = -24 j[0] \land i[0] = -c2021 + j[0] \land 22 = 0\)

Suitable Loop Constraint Solution:
\((23 \rightarrow 22, i[0] \rightarrow -c2021 + j[0], 24 \rightarrow 0)\) with initial value:
\((j[0] \rightarrow 0)\)

Parameter Basis Solution Choice:
\((22 \rightarrow 1)\)

Loop Body:
\((i = 1 + i, j = 1 + j)\)
Initial Value Assignments:
\((i = -c\$2021, j = 0)\)

Assignment of array variable \(z\):
\((z = 1 + n + z)\)

Initial value assignment of array variable \(z\):
\((z = (-1 - c\$2021) n)\)

Corresponding While Loop's
Body: \((i = 1 + i, j = 1 + j, z = 1 + n + z)\)

Condition: \((z < n^2)\)

Initial Value Assignments: \((i = -c\$2021, j = 0, z = (-1 - c\$2021) n)\)

\(\text{QRdecomposition.GetH}\)

\(\text{In}[9]:= \text{loopSynInput} = \{
\{i >= j, \{H[i, j] = QR[i, j]\}\},
\{i < j, \{H[i, j] = 0\}\}
\},
\{i, 0, n\}, \{j, 0, n\}\}
\)

\(\text{Out}[9]= \{(i >= j, \{H[i, j] = QR[i, j]\}, \{i < j, \{H[i, j] = 0\}\}, \{i, 0, n\}, \{j, 0, n\}\}\}
\)

\(\text{In}[10]:= \text{LoopSyn}[\text{loopSynInput}]\)

Starting LoopSyn for 2 loops with matrix indeces \((i, j)\)!

Loop Nr. 1

Guarded assg form: \(((i >= j), \{H[i, j] = QR[i, j]\})\)
Guard: \((i >= j)\)
Guard Polynomial Invariant: \(c\$2122 + i - j\)
Body: \(\{H[i, j] = QR[i, j]\}\)

Creating the loop body with assignments over \((i, j)\)
such that \(c\$2122 + i - j = 0\) is a loop invariant.
Loop condition is \(z < n^2\).

If \('-1 + \&25'\) is a natural number, then:
If \('-1 + \&25'\) is a natural number, then:
Loop Assignments over \((i, j)\):
\[
(i = i + 27 + j \cdot 28, j = j + 26)
\]

Reurrence System:
\[
\]

General Closed Form System:
\[
(i = i[0] + \frac{1}{2} \cdot 27 \cdot (2 \cdot 27 + 28 \cdot ((-1 + 25) \cdot 26 + 2 \cdot j[0])), j = 25 \cdot 26 + j[0])
\]

Branch Guard's Closed Form:
\[
c = 2122 - 25 \cdot 26 + i[0] - j[0] + \frac{1}{2} \cdot 27 \cdot (2 \cdot 27 + 28 \cdot ((-1 + 25) \cdot 26 + 2 \cdot j[0]))
\]

Constraint System on Unknown Coefficients:
\[
(27 \rightarrow 26, i[0] \rightarrow -c2122 + j[0], 28 \rightarrow 0), (27 \rightarrow -28 \cdot j[0], i[0] \rightarrow -c2122 + j[0], 26 \rightarrow 0)
\]

Additional Constraints on the Unknown Coefficients:
\[
(27^2 + 28^2 = 0, 26 \neq 0)
\]

Joint Constraint System:
\[
27 = 26 \& i[0] = -c2122 + j[0] \& 28 = 0
\]

Joint Constraint System:
\[
27 = -28 \cdot j[0] \& i[0] = -c2122 + j[0] \& 26 = 0
\]

Suitable Loop Constraint Solution:
\[
(27 \rightarrow 26, i[0] \rightarrow -c2122 + j[0], 28 \rightarrow 0)
\]

with initial value:
\[
(j[0] \rightarrow 0)
\]

Parameter Basis Solution Choice:
\[
(26 \rightarrow 1)
\]

Loop Body:
\[
(i = 1 + i, j = 1 + j)
\]

Initial Value Assignments:
\[
(i = -c2122, j = 0)
\]

Assignment of array variable \(z\):
\[
(z = 1 + n + z)
\]

Initial value assignment of array variable \(z\):
\[
(z = (-1 - c2122) \cdot n)
\]

Corresponding While Loop's Body:
\[
(i = 1 + i, j = 1 + j, z = 1 + n + z)
\]

Condition:
\[
z < n^2
\]

Initial Value Assignments:
\[
(i = -c2122, j = 0, z = (-1 - c2122) \cdot n)
\]
Loop Nr. 2

Guarded assg form: ((i < j), (H[i, j] = 0))
Guard: (i < j)
Guard Polynomial Invariant: c$2222 + i - j
Body: (H[i, j] = 0)

Creating the loop body with assignments over (i, j)
such that c$2222 + i - j = 0 is a loop invariant.
Loop condition is z < n^2.

If ' 1 + $29' is a natural number, then:

Loop Assignments over (i, j):
(i = i + $31 + j $32, j = j + $30)

Recurrence System:
(i[1 + $29] = $31 + i[$29] + $32 j[$29], j[1 + $29] = $30 + j[$29])

General Closed Form System:
{i = i[0] + 1/2 $29 (2 $31 + $32 ((-1 + $29) $30 + 2 j[0]))}, j = $29 $30 + j[0]}

Branch Guard's Closed Form:
c$2222 - $29 $30 + i[0] - j[0] + 1/2 $29 (2 $31 + $32 ((-1 + $29) $30 + 2 j[0]))

Constraint System on Unknown Coefficients:
((31 \rightarrow 30, i[0] \rightarrow -c$2222 + j[0], \$32 \rightarrow 0), (31 \rightarrow -32 j[0], i[0] \rightarrow -c$2222 + j[0], \$30 \rightarrow 0))

Additional Constraints on the Unknown Coefficients:
($31^2 + 32^2 \neq 0, \$30 \neq 0)

Joint Constraint System:
$31 = $30 \& i[0] = -c$2222 + j[0] \& $32 = 0

Joint Constraint System:
$31 = -32 j[0] \& i[0] = -c$2222 + j[0] \& $30 = 0

Suitable Loop Constraint Solution:
($31 \rightarrow 30, i[0] \rightarrow -c$2222 + j[0], $32 \rightarrow 0)
with initial value:
(j[0] \rightarrow 0)

Parameter Basis Solution Choice:
($30 \rightarrow 1)

Loop Body:
(i = 1 + i, j = 1 + j)
Initial Value Assignments:
\( i = -c2222, \ j = 0 \)

Assignment of array variable \( z \):
\( z = 1 + n + z \)

Initial value assignment of array variable \( z \):
\( z = (-1 - c2222) n \)

Corresponding While Loop's
Body: \( i = 1 + i, j = 1 + j, z = 1 + n + z \)

Condition: \( z < n^2 \)

Initial Value Assignments:
\( i = -c2222, j = 0, z = (-1 - c2222) n \)

\[ \text{Out[10]} = \begin{cases} \{ i = -c62122, j = 0, z = (-1 - c2122) n, \{ z < n^2 \}, \{ i = 1 + i, j = 1 + j, z = 1 + n + z \} \}, \\ \{ i = -c22222, j = 0, z = (-1 - c22222) n, \{ z < n^2 \}, \{ i = 1 + i, j = 1 + j, z = 1 + n + z \} \} \end{cases} \]

\[ \text{In[11]} := \text{loopSynInput} = \{ \] 
  \{ \{ i <= j \}, \{ U[i, j] = LU[i, j] \} \}, 
  \{ \{ i > j \}, \{ U[i, j] = 0 \} \} 
\}, 
\{ \{ i, 0, n \}, \{ j, 0, n \} \} \]

\[ \text{Out[11]} = \{ \{ \{ i <= j \}, \{ U[i, j] = LU[i, j] \} \}, \{ \{ i > j \}, \{ U[i, j] = 0 \} \}, \{ \{ i, 0, n \}, \{ j, 0, n \} \} \}

\[ \text{In[12]} := \text{LoopSyn[loopSynInput]} \]

Starting LoopSyn for 2 loops with matrix indices \( (i, j) \)!

Loop Nr. 1

Guarded assg form: \( \{ i <= j \}, \{ U[i, j] = LU[i, j] \} \)

Guard: \( i <= j \)

Guard Polynomial Invariant: \( c2323 + i - j \)

Body: \( U[i, j] = LU[i, j] \)

Creating the loop body with assignments over \( (i, j) \)
such that \( c2323 + i - j = 0 \) is a loop invariant.
Loop condition is \( z < n^2 \).

If `-1 + $33$' is a natural number, then:

If `-1 + $33$' is a natural number, then:

Loop Assignments over \( (i, j) \):
\( i = i + 35 + j 36, j = j + 34 \)
Recurrence System:
\[ \{ i[1 + i] = j[1 + j], j[1 + i] = j[1 + j] \} \]

General Closed Form System:
\[ \{ i = i[0] + 1, j = j[0] + 1 \} \]

Branch Guard's Closed Form:
\[ c = j[0] + j[0] + 1 \]

Constraint System on Unknown Coefficients:
\[ \{ j[1 + j], j[1 + j] = j[1 + j] \} \]

Additional Constraints on the Unknown Coefficients:
\[ \{ j[1 + j], j[1 + j] \} \]

Suitable Loop Constraint Solution:
\[ \{ j[1 + j], j[1 + j] \} \]

Initial Value Assignments:
\[ \{ i = 1, j = 1 \} \]

Assignment of array variable z:
\[ z = 1 + n + z \]

Initial value assignment of array variable z:
\[ z = (-1 - c) \]

Corresponding While Loop's Body:
\[ (i = 1 + i, j = 1 + j, z = 1 + n + z) \]

Condition: \( z < n^2 \)

Initial Value Assignments:
\[ (i = 1, j = 1, z = (-1 - c) n) \]
Guarded assg form: \([\{i > j\}, (U[i, j] = 0)]\)
Guard: \((i > j)\)
Guard Polynomial Invariant: \(c\cdot 2423 + i - j\)
Body: \((U[i, j] = 0)\)

Creating the loop body with assignments over \((i, j)\)
such that \(c\cdot 2423 + i - j = 0\) is a loop invariant.
Loop condition is \(z < n^2\).

If "' - 1 + $37' is a natural number, then:

If "' - 1 + $37' is a natural number, then:

Loop Assignments over \((i, j)\):
\(i = i + 39 + j \cdot 40, j = j + 38\)

Recurrence System:
\(((i[1 + 37] = 39 + i[37] + 40 \cdot j[37], j[1 + 37] = 38 + j[37]))\)

General Closed Form System:
\(\{i = i[0] + \frac{1}{2} \cdot 37 \cdot (2 \cdot 39 + 40 \cdot (-1 + 37) \cdot 38 + 2 \cdot j[0])), j = 37 \cdot 38 + j[0]\}\)

Branch Guard's Closed Form:
\(c\cdot 2423 - 37 \cdot 38 + i[0] - j[0] + \frac{1}{2} \cdot 37 \cdot (2 \cdot 39 + 40 \cdot (-1 + 37) \cdot 38 + 2 \cdot j[0]))\)

Constraint System on Unknown Coefficients:
\(\{(39 \rightarrow 38, i[0] \rightarrow -c\cdot 2423 + j[0], 40 \rightarrow 0), (39 \rightarrow -40 \cdot j[0], i[0] \rightarrow -c\cdot 2423 + j[0], 38 \rightarrow 0)\}\)

Additional Constraints on the Unknown Coefficients:
\(\{39^2 + 40^2 \neq 0, 38 \neq 0\}\)

Joint Constraint System:
\(39 = 38 \& i[0] = -c\cdot 2423 + j[0] \& 40 = 0\)

Joint Constraint System:
\(39 = -40 \cdot j[0] \& i[0] = -c\cdot 2423 + j[0] \& 38 = 0\)

Suitable Loop Constraint Solution:
\(\{(39 \rightarrow 38, i[0] \rightarrow -c\cdot 2423 + j[0], 40 \rightarrow 0)\}\)
with initial value:
\(\{j[0] \rightarrow 0\}\)

Parameter Basis Solution Choice:
\(\{38 \rightarrow 1\}\)

Loop Body:
\(\{i = 1 + i, j = 1 + j\}\)
Initial Value Assignments:
\((i = -c\$2423, j = 0)\)

Assignment of array variable \(z\):
\((z = 1 + n + z)\)

Initial value assignment of array variable \(z\):
\((z = (-1 - c\$2423) n)\)

Corresponding While Loop's

Body:
\((i = 1 + i, j = 1 + j, z = 1 + n + z)\)

Condition: \((z < n^2)\)

Initial Value Assignments:
\((i = -c\$2423, j = 0, z = (-1 - c\$2423) n)\)

\(Out[12] = \{(i = -c\$2423, j = 0, z = (-1 - c\$2423) n), \{z < n^2\}, \{i = 1 + i, j = 1 + j, z = 1 + n + z\}\}, \{(i = -c\$2423, j = 0, z = (-1 - c\$2423) n), \{z < n^2\}, \{i = 1 + i, j = 1 + j, z = 1 + n + z\}\}\)

Aligators

\(In[1]:= \text{\textless\textless Aligators.m}\)

\begin{verbatim}
Aligators.m
Aligator on Arrays.
Package written by Laura Kovacs — © EPFL Lausanne — V 0.3 (2009-01-29)

Dependencies.m by Manuel Kauers and Burkhard Zimmermann. Changed by Laura Kovacs for
Invariant Generation — © RISC Linz — V 0.27 (2008-02-07)

Fast Zeilberger Package by Peter Paule, Markus Schorn, and Axel Riese — © RISC Linz — V 3.52 (01/12/05)
\end{verbatim}
Loop 1

In[11] := Aligators[WHILE[z < n^2, i = 1 + i; j = 1 + j; z = 1 + n + z; L[z] = LUA[(1 - 1) * n + j]], IniVal → {i = c; j = n; z = c * n - n}]

ALIGATORS version with SIMULTANEOUS Assignments:

Loop Recurrence System:
\[
\begin{align*}
[i[1 + \$18] = 1 + i[\$18], j[1 + \$18] = 1 + j[\$18], \\
z[1 + \$18] = 1 + n + z[\$18], L[1 + \$18][z[\$18]] &= LUA[n (-1 + i[\$18]) + j[\$18]]
\end{align*}
\]

If `-1 + \$18` is a natural number, then:

If `-1 + \$18` is a natural number, then:

If `-1 + \$18` is a natural number, then:

Full indexed scalar recurrence system:
\[
\begin{align*}
[i[\$18] &= \$18 + i[0], j[\$18] &= \$18 + j[0], z[\$18] &= (1 + n) \$18 + z[0]
\end{align*}
\]

Full indexed array recurrence system:
\[
\begin{align*}
[L[\$18][(1 + n) (-1 + \$18) + z[0]] &= LUA[-1 + \$18 + n (-2 + \$18 + i[0]) + j[0]]
\end{align*}
\]

Condition vars: \((n, z)\)

Commutable Loop: \((z = 1 + n + z)\)

If `-1 + \$19` is a natural number, then:

Full indexed scalar recurrence system:
\[
\begin{align*}
z[\$19] &= (1 + n) \$19 + z[0]
\end{align*}
\]

Full indexed array recurrence system:
\[
\]

Bound: True

Out[11] := \(V_{k\$2520,0-k\$2520,0}^{n-cn} (L[-n + c n + (-1 + k\$2520) (1 + n)] = \\
LUA[-1 + k\$2520 + n + (-2 + c + k\$2520) n] \& \& -n + c n + (-1 + k\$2520) (1 + n) < n^2) \& \&
\]
\[
i + n = c + j \& \& j - n + c n + j n = n + n^2 + z
\]
Loop 2

\[ n[12] := \text{Aligators[WHILE}[z < n^2, i = 1 + i; j = 1 + j; z = 1 + n + z; L[z] = 1], IniVal \rightarrow (i = 0; j = 0; z = -n)] \]

ALIGATORS version with SIMULTANEOUS Assignments:

Loop Recurrence System:
\[
\begin{align*}
(i[1 + %2] &= 1 + i[%2], j[1 + %2] &= 1 + j[%2], z[1 + %2] &= 1 + n + z[%2], L[1 + %2][z[%2]] = 1)
\end{align*}
\]

If \(-1 + %2\)' is a natural number, then:

Full indexed scalar recurrence system:
\[
\begin{align*}
(i[%2] &= %2 + i[0], j[%2] &= %2 + j[0], z[%2] &= (1 + n) %2 + z[0])
\end{align*}
\]

Full indexed array recurrence system:
\[
\begin{align*}
(L[%2][(1 + n)(-1 + %2) + z[0]] = 1)
\end{align*}
\]

Condition vars: \((n, z)\)

Commutable Loop: \((z = 1 + n + z)\)

If \(-1 + %2\) is a natural number, then:

Full indexed scalar recurrence system:
\[
\begin{align*}
(z[%2] &= (1 + n) %2 + z[0])
\end{align*}
\]

Full indexed array recurrence system:
\[
\begin{align*}
()\]

Bound: True

\[ Out[12] = \forall k_{2777}.0 < k_{2777}.0 \Rightarrow (L[-n + (-1 + k_{2777} \cdot 1 + n)] = 1 \&\& -n + (-1 + k_{2777} \cdot 1 + n) < n^2) \&\& i = j \&\& j - n + jn = z \]
## Loop 3

In[13]:= Aligators[WHILE[z < n^2, \(i = 1 + i; j = 1 + j; z = 1 + n + z; L[z] = 0\)], IniVal \to (i = 0; j = c; z = c - n)]

ALIGATORS version with SIMULTANEOUS Assignments:

Loop Recurrence System:
\[
\begin{align*}
i[1 + \$26] &= 1 + i[\$26], \\
j[1 + \$26] &= 1 + j[\$26], \\
z[1 + \$26] &= 1 + n + z[\$26], \\
L[1 + \$26][z[\$26]] &= 0
\end{align*}
\]

If `-1 + $26' is a natural number, then:

If `-1 + $26' is a natural number, then:

If `-1 + $26' is a natural number, then:

Full indexed scalar recurrence system:
\[
\begin{align*}
i[\$26] &= \$26 + i[0], \\
j[\$26] &= \$26 + j[0], \\
z[\$26] &= (1 + n) \$26 + z[0]
\end{align*}
\]

Full indexed array recurrence system:
\[
\begin{align*}
L[\$26][i[\$26] \{1 + n\} (-1 + \$26) + z[0]] &= 0
\end{align*}
\]

Condition vars: (n, z)

Commuatable Loop: \((z = 1 + n + z)\)

If `-1 + $27' is a natural number, then:

Full indexed scalar recurrence system:
\[
\begin{align*}
z[\$27] &= (1 + n) \$27 + z[0]
\end{align*}
\]

Full indexed array recurrence system:
\[
\begin{align*}
\end{align*}
\]

Bound: True

Out[13]= \[\forall_{k = 3017, 0 < k < 3017}(c - n + (-1 + k \times 3017) \{1 + n\} = 0 \&\& c - n + (-1 + k \times 3017) \{1 + n\} < n^2) \&\& c + i = j \&\& c + j - n + j n = c + c n + z\]