Problem 3

There are $n \geq 1$ asynchronous processes, each with a private memory of p bits. The processes can communicate only through reading and writing a shared memory of s bits. Computation proceeds in a sequence of atomic steps: in each step, first a scheduler chooses one of the processes; then the chosen process reads and modifies his private state and the shared state. A *fair* scheduler does not neglect any process forever; a *bounded fair* scheduler does not neglect any process for more than b steps, for some integer bound $b \geq n$.

You need to design a protocol that allows the processes to elect a leader. In addition to the private memory of p bits, each process i has a read-only variable F_i and a write-only variable V_i. The variable $F_i \in \{1, \ldots, n\}$ represents the initial favorite of process i for leader. The variable $V_i \in \{1, \ldots, n\}$ represents the final vote of process i for leader. In addition to the shared memory of s bits, there is a shared write-only bit T, which is initially 0. Setting T to 1 signals the completion of the election process. The requirements on the protocol are as follows:

1. Leader election must be unanimous, that is, when $T = 1$, then $V_i = V_j$ for all $1 \leq i, j \leq n$.

2. One of the initial favorites must be elected leader, that is, when $T = 1$, then $|\{j \mid F_j = V_1\}| \geq |\{j \mid F_j = i\}|$ for all $1 \leq i \leq n$.

3. Leader election must terminate, that is, if the scheduler is fair, then $T = 1$ within a finite number of steps.

Each process can initialize his private memory (with the exception of the F_i's), but the initial state of the shared memory (with the exception of T) is unknown. Each process knows the number n of processes, but is oblivious to the choices made by the scheduler.

Questions:

1. What is the smallest number s of shared bits necessary to solve the problem?

2. Given a solution, for a scheduler with bound b, let t_b be the maximal number of steps till termination ($T = 1$), and let $s \cdot t_b$ be the *cost* of the solution. What is the least cost necessary to solve the problem for bounded schedulers with unknown bound b?