SCRIBE NOTES - October 9, 2008

Contest Results for SSG Project

Experiment	Nodes	GRP1	GRP2	GRP4	GRP5	GRP7	Highest 3 Random Nodes
Contest 2	1000	30	6	3	6	6	139, 34, 3
Contest 3	1500	?	10	4	7	8	440, 307, 46
Contest 4	2000	10	8	3	7	7	317, 302, 186
Contest 5	3000	54	7	5	7	9	277(0.72), 1224(0.71), 131(0.7)
Group 2	500	9	7	2	7	4	65(0.85), 30(0.86), 99(0.86)
Note: Group 4 contest was omitted as the graph was Non Stopping							

Verena's Algorithm

Input: P - transition matrix, t - type vector
//compute attractor set of node T_{max} and change P accordingly
$(P, W) := Attr\langle T_{max}, P, t\rangle;$
$T := P;$
//compute set Q of all nodes that are
//1) random nodes and can reach W via random nodes or 2) in W
$Q :=$ BFSrand(W);
//compute probabilities of reaching T_{max} in subgraph Q
$X =$ Solve(T, t, Q);
//sort X in decreasing order and store indices
indices = Sort(X),
while Q does not change
  for $i = 1$: length(X)
   u :=indices(i);
   if($t(u)$ == RandomNode)
    $(T, U) := Attr\langle u, T, t\rangle;$
    $Q := Q \cup$ BFSrand(U);
   end
  end
  $X :=$ Solve(T, t, Q);
  indices:= Sort(X);
  T:=P;
end

Claim 1: On termination the permutation (w.r.t X) is self consistent.
Claim 2: The number of iterations of the loop is polynomial in size of graph.

Algorithm For Generating Random SSGs

$Q_{min} = \{T_{min}\}$, $Q_{max} = \{T_{max}\}$, N = number of nodes

//First Round
Level(T_{max}):=0; Level(T_{min}):=0;
For all nodes v

    either with probability 1/2:
        find left successor w in Q_{min};
        $Q_{min} = Q_{min} \cup \{v\}$
        Level(v) = Level(w) + 1;

    otherwise:
        find left successor w in Q_{max};
        $Q_{max} = Q_{max} \cup \{v\}$
        Level(v) = Level(w) + 1;

//Second Round
For each min/max node,
choose randomly a right successor with smaller level

For each random node,
choose right successor arbitrarily

Kelley Topology

Two sets A and B are *equipollent* iff there is a bijective function from A to B.

Example
$A = \mathbb{N}$, B = set of even numbers
$f : A \to B$ with $f(n) = 2n$ is bijective

Theorem 20
If there is a one-to-one function from A to B, and there is a one-to-one function from B to A, then A and B are equipollent.

Definition one-to-one
f is *one-to-one* if for all $x, y \in A$, if $x \neq y$, then $f(x) \neq f(y)$.

Definition onto
$f : A \to B$ is *onto* (range B), if for all $y \in B$, there exists $x \in A$, such that $f(x) = y$.

Definition bijection
f is a *bijection* if it is one-to-one and onto.