1 Humans and Monkeys

For homework, we had to prove the following statement.

**Theorem.** If there is a human with an ancestor that is a monkey, then there is a human with a parent that is a monkey.

Let us first formalize the theorem in first-order logic using the predicate $h(x)$ to denote “$x$ is a human” and $m(x)$ to denote “$x$ is a monkey”:

$$(\forall x)(\exists y)(h(x) \land m(y) \land \text{anc}(y, x)) \Rightarrow (\exists u, w)(h(u) \land m(w) \land w \prec u).$$

**Proof.** Consider an arbitrary $\hat{x}$. We formulate the following induction hypothesis:

$$(\forall z)(z \prec \hat{x} \Rightarrow (\exists y)(h(z) \land m(y) \land \text{anc}(y, z)) \Rightarrow (\exists u, w)(h(u) \land m(w) \land w \prec u)).$$

Show:

$$(\exists y)h(\hat{x}) \land m(y) \land \text{anc}(y, \hat{x}) \Rightarrow (\exists u, w)(h(u) \land m(w) \land w \prec u).$$

Let $\hat{y}$ be such that $h(\hat{x}, m(\hat{y}))$, and $\text{anc}(\hat{y}, \hat{x})$. Show $(\exists u, w)(h(u) \land m(w) \land w \prec u)$.

- **Case 1**: $\hat{y} \prec \hat{x}$. Choose $u = \hat{x}, w = \hat{y}$.

- **Case 2**: $(\exists t)(\text{anc}(\hat{y}, t) \land t \prec \hat{x})$. Let $\hat{t}$ be such that $\text{anc}(\hat{y}, \hat{t}), \hat{t} \prec \hat{x}$.

  - **Case 2.1**: $m(\hat{t})$. Choose $u = \hat{x}, w = \hat{t}$.

  - **Case 2.2**: $h(\hat{t})$. By induction hypothesis,

    $$(\exists y)(h(\hat{t}) \land m(y) \land \text{anc}(y, \hat{t})) \Rightarrow (\exists u, w)(h(u) \land m(w) \land w \prec u).$$

    Since $h(\hat{t}) \land m(\hat{y}) \land \text{anc}(\hat{y}, \hat{t})$, we know $(\exists u, w)(h(u) \land m(w) \land w \prec u)$. 

\qed
2 Reports on Project 1

The reports on Project 1 (Reliability Calculus) are due on Tuesday, April 17. Each report should contain the following elements.

- A formal definition of \( s, \mu \models \phi \).
- An informal or formal justification of the above definition explaining in particular:
  - the model of computation on the component level (which sequence of values does a component compute?), e.g.: If each function \( f_i \) computes
    \[
    \llbracket f_i \rrbracket : \llbracket R(f_i) \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket \rightarrow \llbracket W(f_i) \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket,
    \]
    then the component \( \phi \) computes
    \[
    \llbracket \phi \rrbracket : \llbracket I \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket \rightarrow \llbracket O \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket \ldots
    \]
  - the model of computation on the platform level (which sequence of values does a component mapped to a platform compute?), e.g.: If each function \( f_i \) computes
    \[
    \llbracket f_i \rrbracket : \llbracket R(f_i) \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket \rightarrow \llbracket W(f_i) \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket,
    \]
    then the component \( \phi \) mapped by \( \mu \) to the platform \( s \) computes
    \[
    \llbracket \phi, \mu, s \rrbracket : \llbracket I \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket \rightarrow \llbracket O \rightarrow \mathbb{N} \rightarrow \mathbb{N} \rrbracket \ldots
    \]
  - the model of reliability.
- An algorithm.
- A theorem and its proof.

3 Description of Project 2: \( h \)-number

The second project will be about the \( h \)-number (also called \( h \)-index). This is a measure of scientific output based on the number of publication a researcher has written and the number of citation each of these publications has received. For instance, a \( h \)-number of 20 means that a scientist has 20 publications that are each cited at least by 20 other publications. On the handout we have a script of Michael Schwartzbach which computes the \( h \)-number for an individual scientist by querying Google Scholar.

The aim of the project is to find an automated way of measuring the output of scientific communities (e.g. Computer-Science departments of universites, conference participants, countries) rather than of individual researchers.