7 Proofs, Setting up Project 2

Scribe: Valentin Longchamp 3. April 2007

1 Humans and Monkeys

For homework, we had to prove the following statement.

> **Theorem.** If there is a human with an ancestor that is a monkey, then there is a human with a parent that is a monkey.

Let us first formalize the theorem in first-order logic using the predicate $h(x)$ to denote "x is a human" and $m(x)$ to denote "x is a monkey":

$$(\forall x)(\exists y)(h(x) \wedge m(y) \wedge \text{anc}(y, x)) \Rightarrow (\exists u, w)(h(u) \wedge m(w) \wedge w \prec u).$$

Proof. Consider an arbitrary $\hat{x}$. We formulate the following induction hypothesis:

$$(\forall z)(z \prec \hat{x} \Rightarrow (\exists y)(h(z) \wedge m(y) \wedge \text{anc}(y, z)) \Rightarrow (\exists u, w)(h(u) \wedge m(w) \wedge w \prec u).$$

Show:

$$(\exists y)h(\hat{x}) \wedge m(y) \wedge \text{anc}(y, \hat{x}) \Rightarrow (\exists u, w)(h(u) \wedge m(w) \wedge w \prec u).$$

Let $\hat{y}$ be such that $h(\hat{x}$, $m(\hat{y})$, and $\text{anc}(\hat{y}, \hat{x})$. Show $(\exists u, w)(h(u) \wedge m(w) \wedge w \prec u)$.

- *Case 1*: $\hat{y} \prec \hat{x}$. Choose $u = \hat{x}, w = \hat{y}$.
- *Case 2*: $(\exists t)(anc(\hat{y}, t) \wedge t \prec \hat{x})$. Let $\hat{t}$ be such that $anc(\hat{y}, \hat{t}), \hat{t} \prec \hat{x}$.
  - *Case 2.1*: $m(\hat{t})$. Choose $u = \hat{x}, w = \hat{t}$.
  - *Case 2.2*: $h(\hat{t})$. By induction hypothesis,

    $$(\exists y)(h(\hat{t}) \wedge m(y) \wedge anc(y, \hat{t})) \Rightarrow (\exists u, w)(h(u) \wedge m(w) \wedge w \prec u).$$

    Since $h(\hat{t}) \wedge m(\hat{y}) \wedge \text{anc}(\hat{y}, \hat{t})$, we know $(\exists u, w)(h(u) \wedge m(w) \wedge w \prec u)$.

□

2 Reports on Project 1

The reports on Project 1 (Reliability Calculus) are due on Tuesday, April 17. Each report should contain the following elements.

- A formal definition of $s, \mu \models \phi$.
- An informal or formal justification of the above definition explaining in particular:
  - the model of computation on the component level (which sequence of values does a component compute?), e.g.: If each function f_i computes

    $$[\![f_i]\!] : [R(f_i) \to \mathbb{N} \to \mathbb{N}] \to [W(f_i) \to \mathbb{N} \to \mathbb{N}],$$

    then the component ϕ computes

    $$[\![\phi]\!] : [I \to \mathbb{N} \to \mathbb{N}] \to [O \to \mathbb{N} \to \mathbb{N}] \ldots$$

  - the model of computation on the platform level (which sequence of values does a component mapped to a platform compute?), e.g.: If each function f_i computes

    $$[\![f_i]\!] : [R(f_i) \to \mathbb{N} \to \mathbb{N}] \to [W(f_i) \to \mathbb{N} \to \mathbb{N}],$$

    then the component ϕ mapped by μ to the platform s computes

    $$[\![\phi, \mu, s]\!] : [I \to \mathbb{N} \to \mathbb{N}] \to [O \to \mathbb{N} \to \mathbb{N}] \ldots$$

  - the model of reliability.
- An algorithm.
- A theorem and its proof.

3 Description of Project 2: h-number

The second project will be about the *h-number* (also called h-index). This is a measure of scientific output based on the number of publication a researcher has written and the number of citation each of these publications has received. For instance, a h-number of 20 means that a scientist has 20 publications that are each cited at least by 20 other publications. On the handout we have a script of Michael Schwartzbach which computes the h-number for an individual scientist by querying Google Scholar.

The aim of the project is to find an automated way of measuring the output of scientific communities (e.g. Computer-Science departments of universites, conference participants, countries) rather than of individual researchers.